Identifying topological phases in real space

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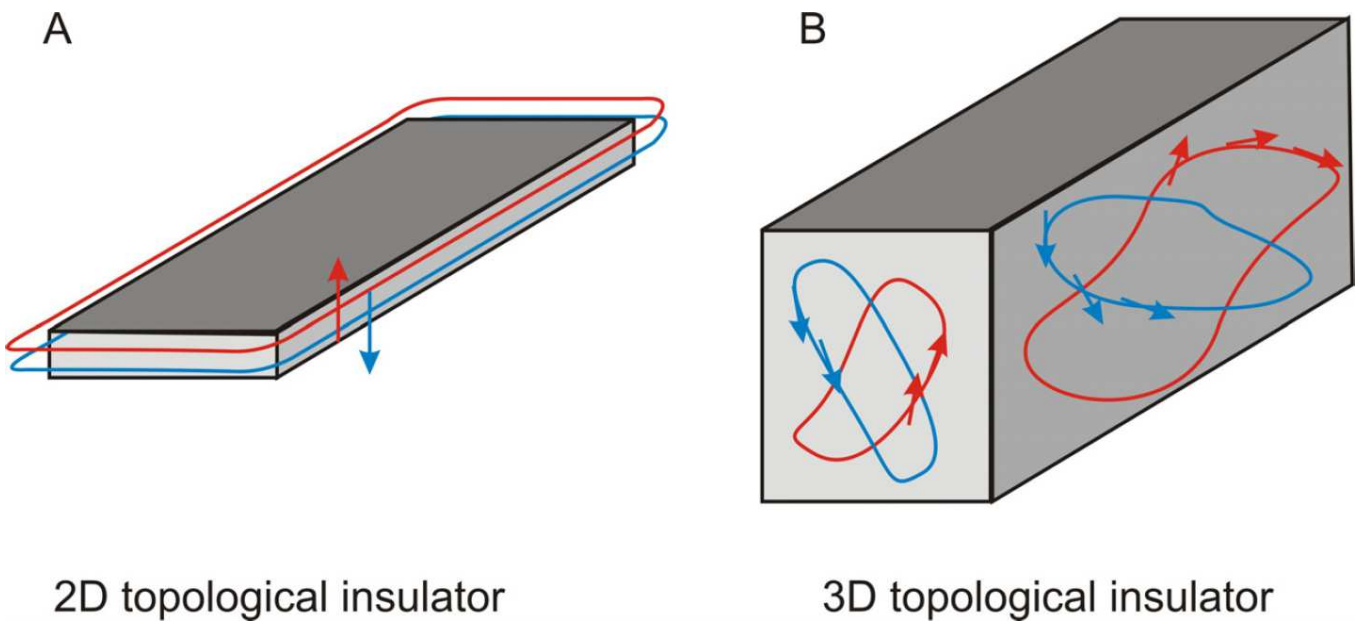
Supported by Office of Naval Research and by JQI-NSF-PFC

2010 APS March Meeting

Topological Insulators

- Bulk insulating gap
- Edges (2D) or surfaces (3D) support gapless transport
- Cannot be simply transformed to destroy the surface states

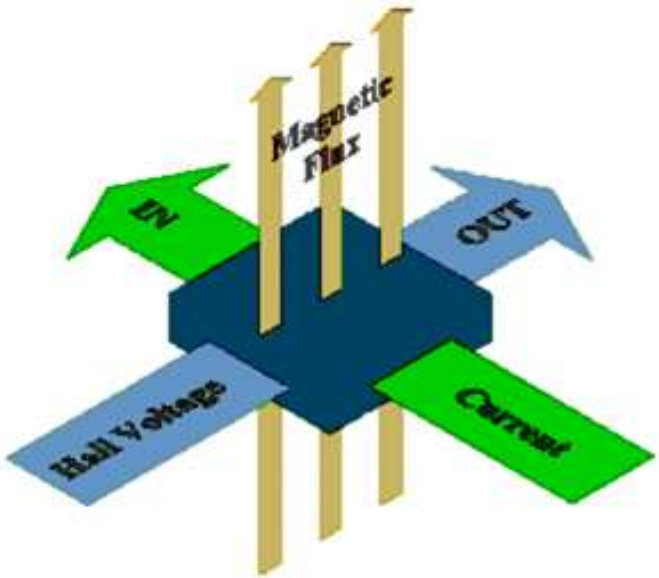


Stanford Science Highlight March 2009

Hall Effect

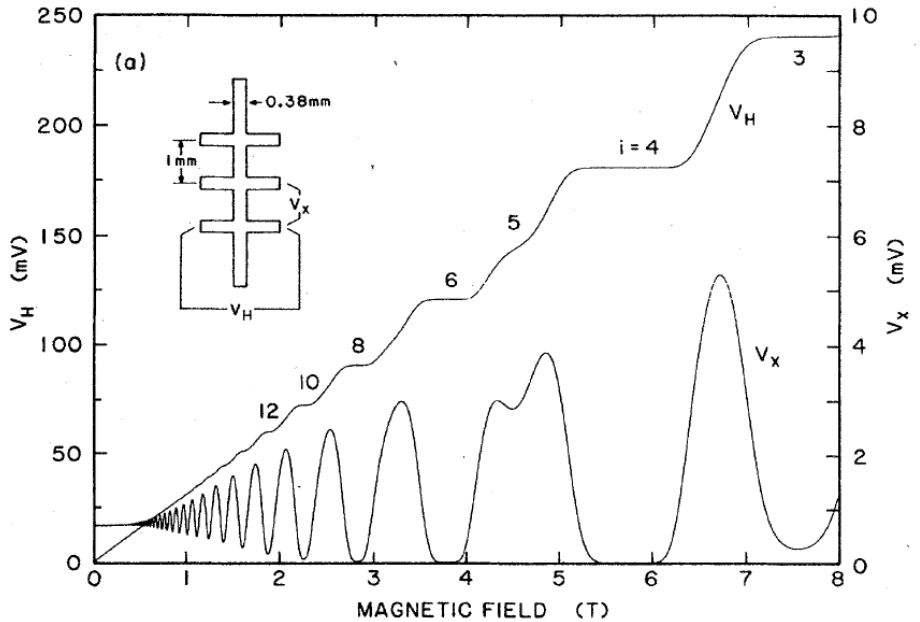
Hall Effect:

- A magnetic field B applied perpendicular to a current I causes a voltage difference perpendicular to both B and I.



Quantum Hall Effect:

- Hall conductivity is quantized



Cage *et al.* IEEE Trans. Instrum. Meas. **IM-34**, 301 (1985).

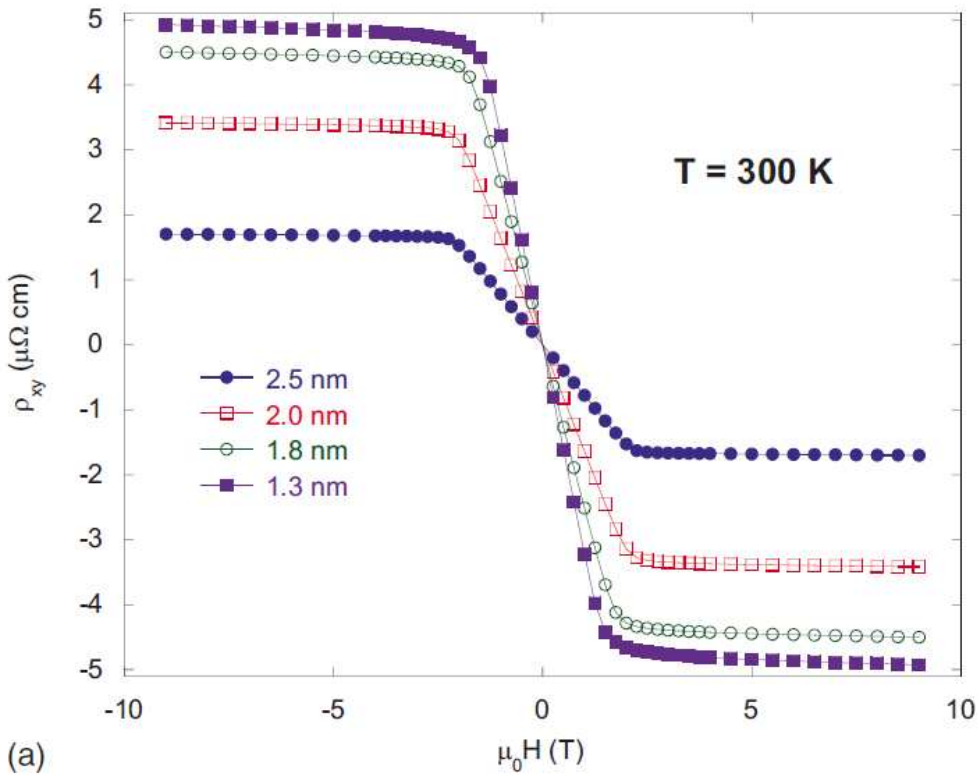
Anomalous Hall Effect

Anomalous Hall Effect (AHE):

- No net magnetic field
- Non-zero Hall conductivity σ_{xy}
- Break Chiral symmetry
- Metallic state

Quantum AHE:

- σ_{xy} quantized
- Insulating state



Sangiao *et al.*, PRB **79**, 014431 (2009).

Nice review of AHE: Nagaosa *et al.*, arXiv:0904.4154

Questions

Challenge:

- Want to study systems with open boundaries
- No local order parameter
- Cannot calculate Chern number

Questions:

- How do we characterize topological order in real space?
- What do the edge states look like?
- What role does the type of edge play?

Model / Measurements

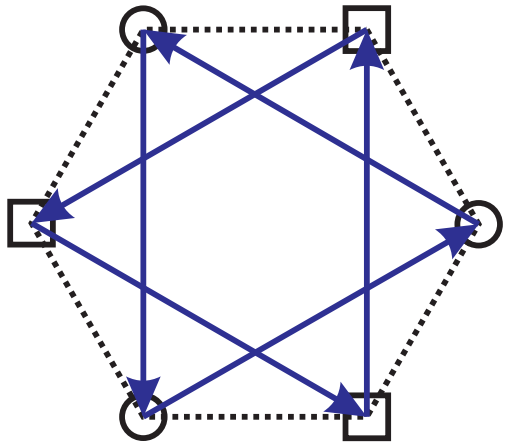
Haldane Model:

$$H = -\sum_{\langle ij \rangle} \left(t_1 c_i^\dagger c_j + \text{h.c.} \right) - \sum_{\langle\langle ij \rangle\rangle} \left(t_2 e^{i\phi_{ij}} c_i^\dagger c_j + \text{h.c.} \right) + M \sum_i (-1)^{\sigma(i)} n_i$$

Current:

$$\mathbf{J}_{ij} = \frac{iq}{\hbar} \mathbf{r}_{ij} \left(t_{ij} c_i^\dagger c_j - t_{ji} c_j^\dagger c_i \right)$$

Haldane:



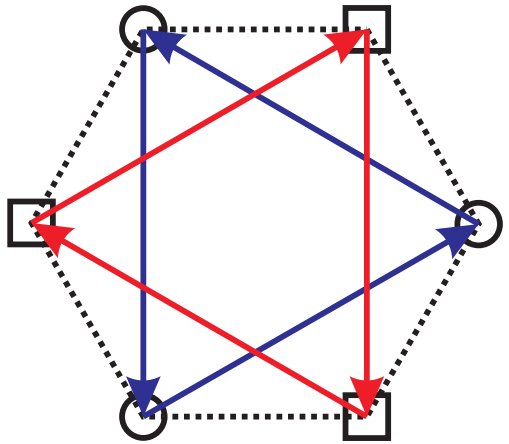
t_1 .....

$t_2 e^{i\phi}$ →

$+M$ ○

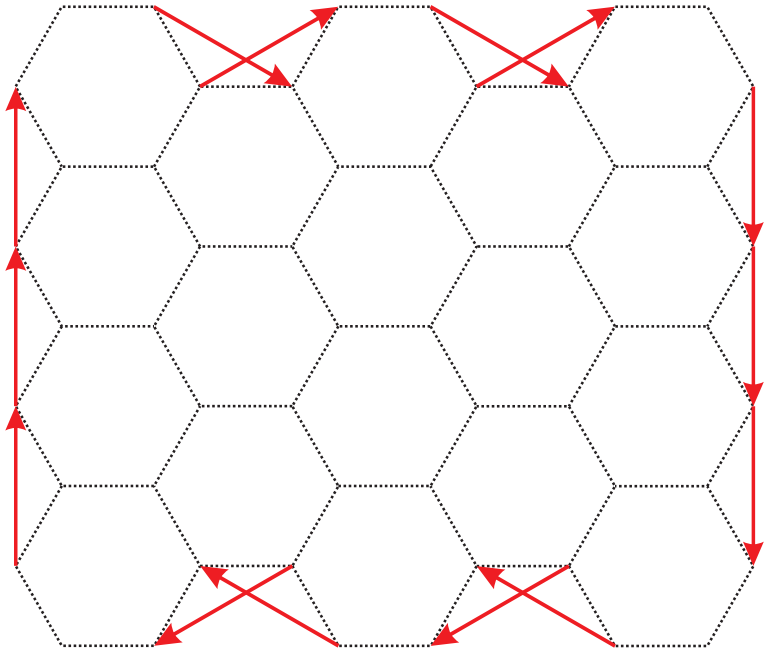
$-M$ □

Modified:



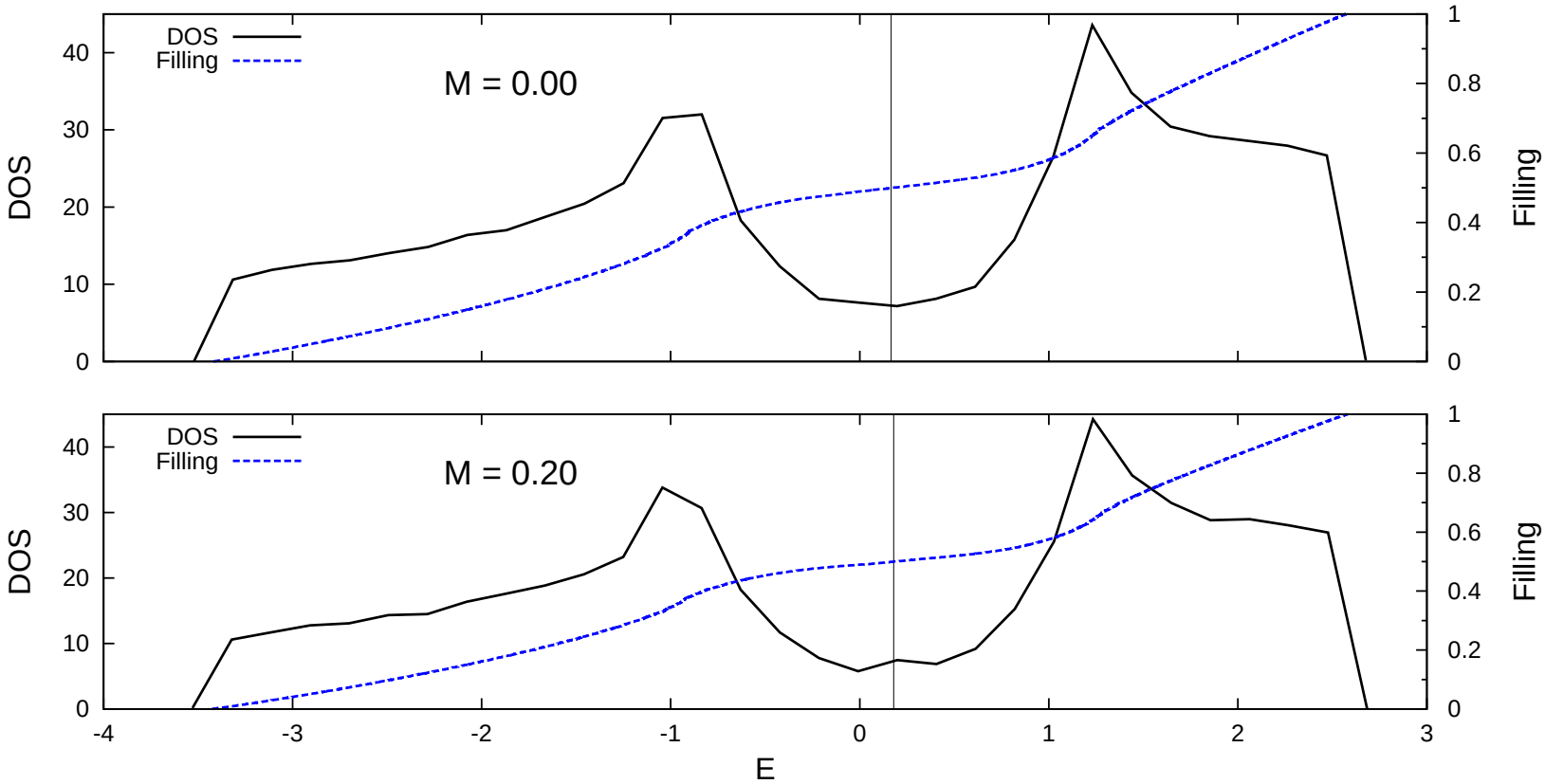
Treating the edge

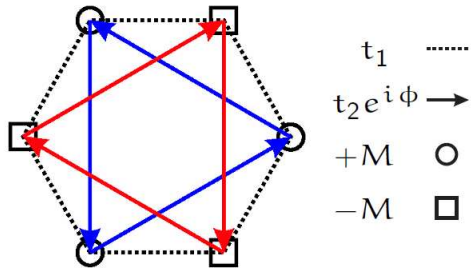
- Boundary next-nearest-neighbor hoppings cause additional flux



- Type of edge (armchair vs. zigzag) affects boundary current.
- Must remove these terms to characterize topological order.

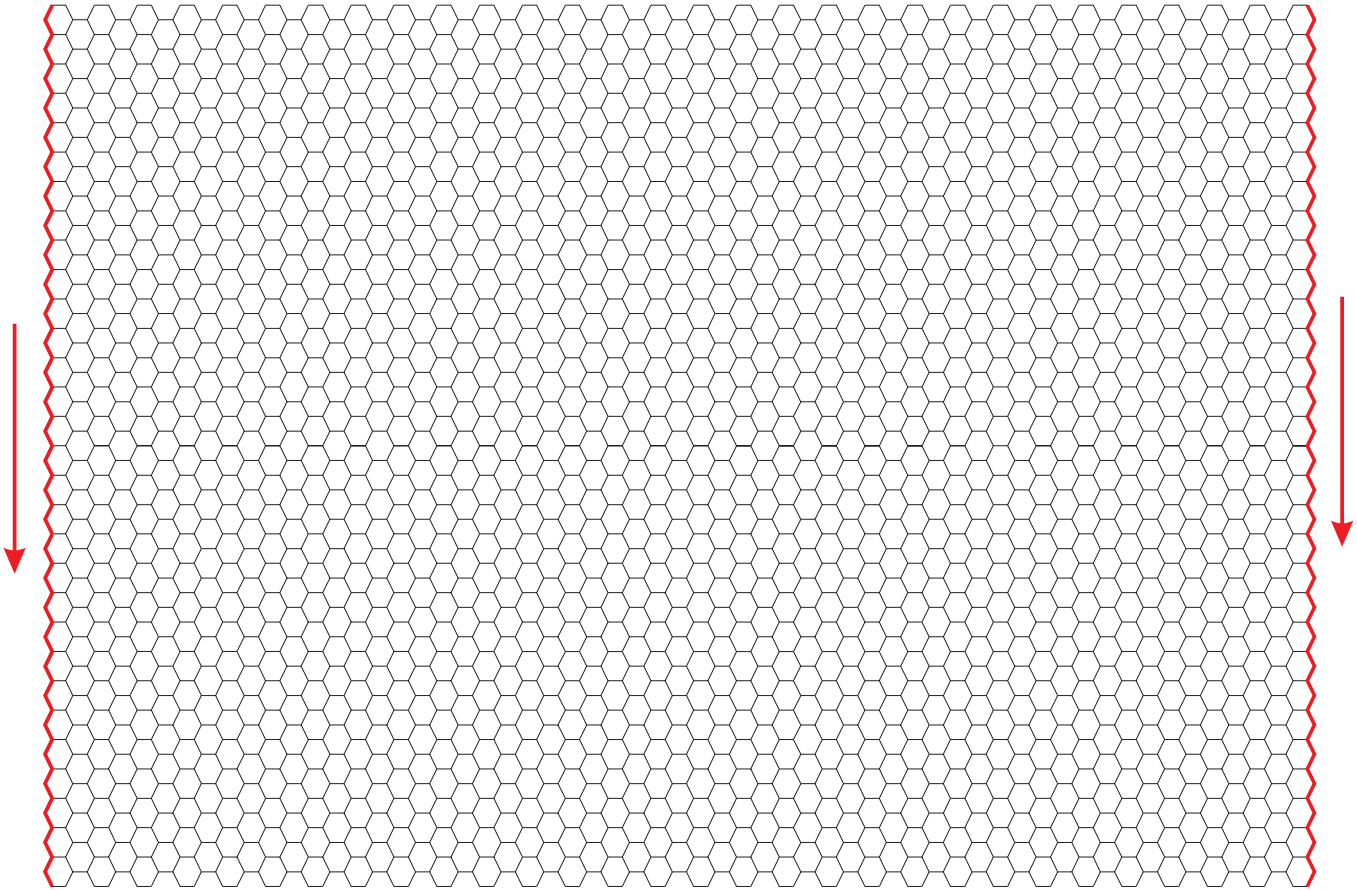
Modified (60×61, $t_2 = 0.1t_1$, $\phi = \pi/4$)





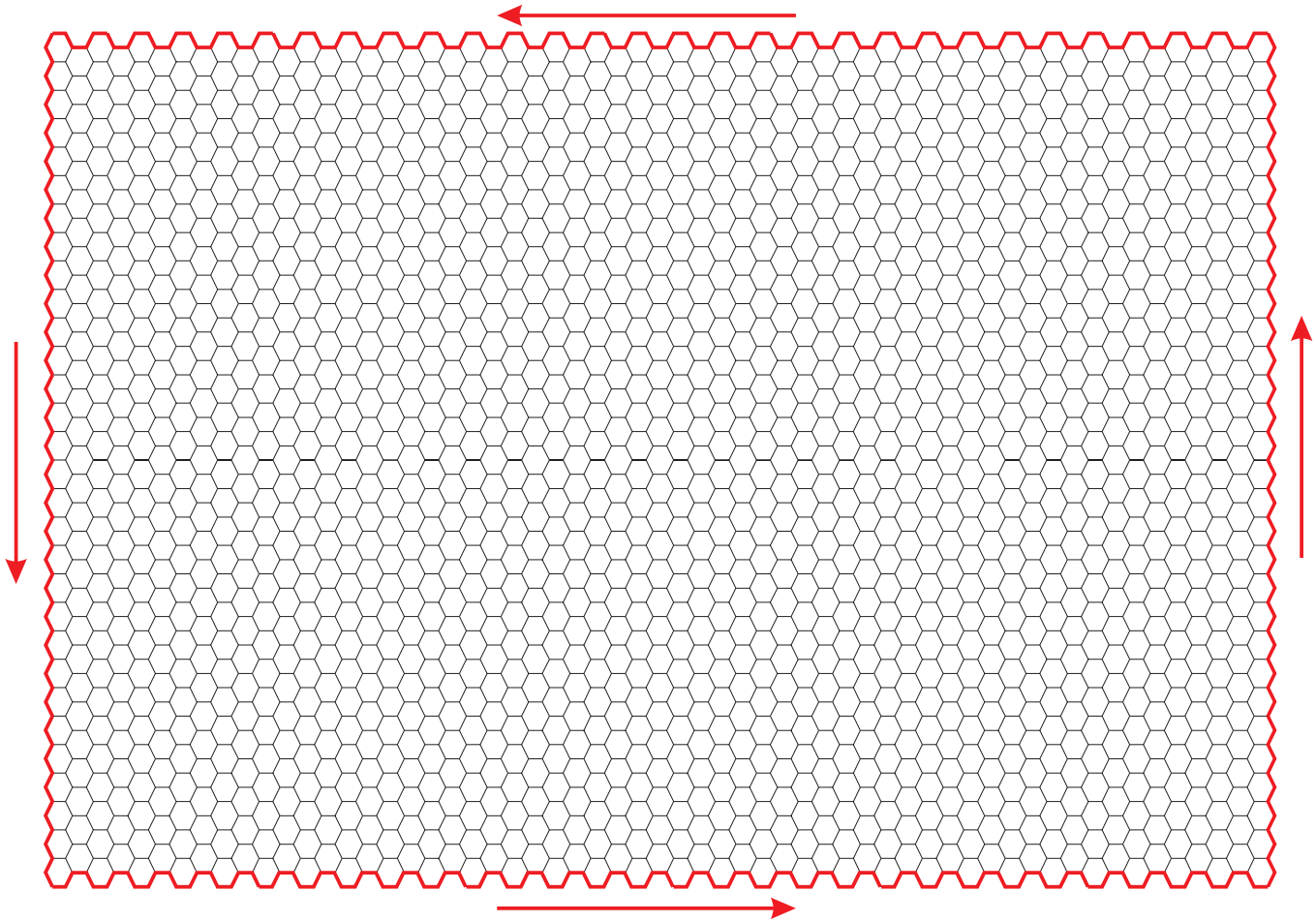
Modified $(60 \times 61,\ t_2 = 0.1t_1,\ \phi = \pi/4)$

$M = 0.00$

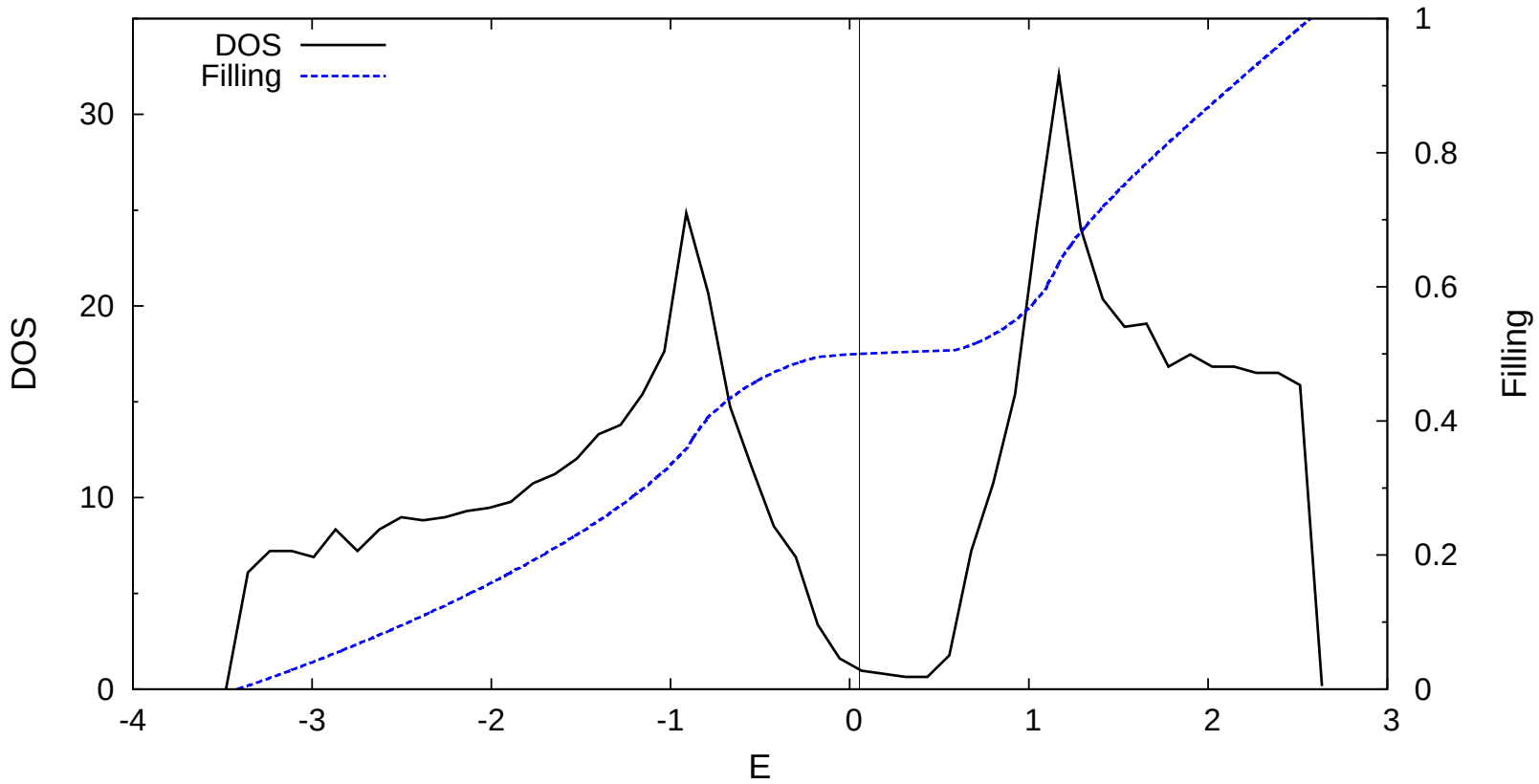


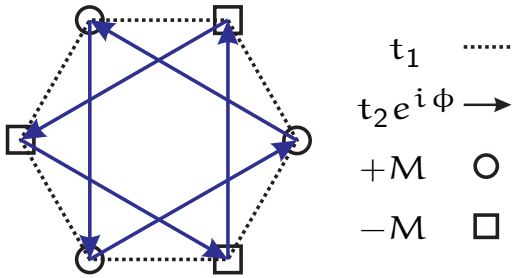
Modified $(60 \times 61,\ t_2 = 0.1t_1,\ \phi = \pi/4)$

$M = 0.20$



Haldane (60×61, $t_2 = 0.1t_1$, $\phi = \pi/4$, $M = 0.00$)





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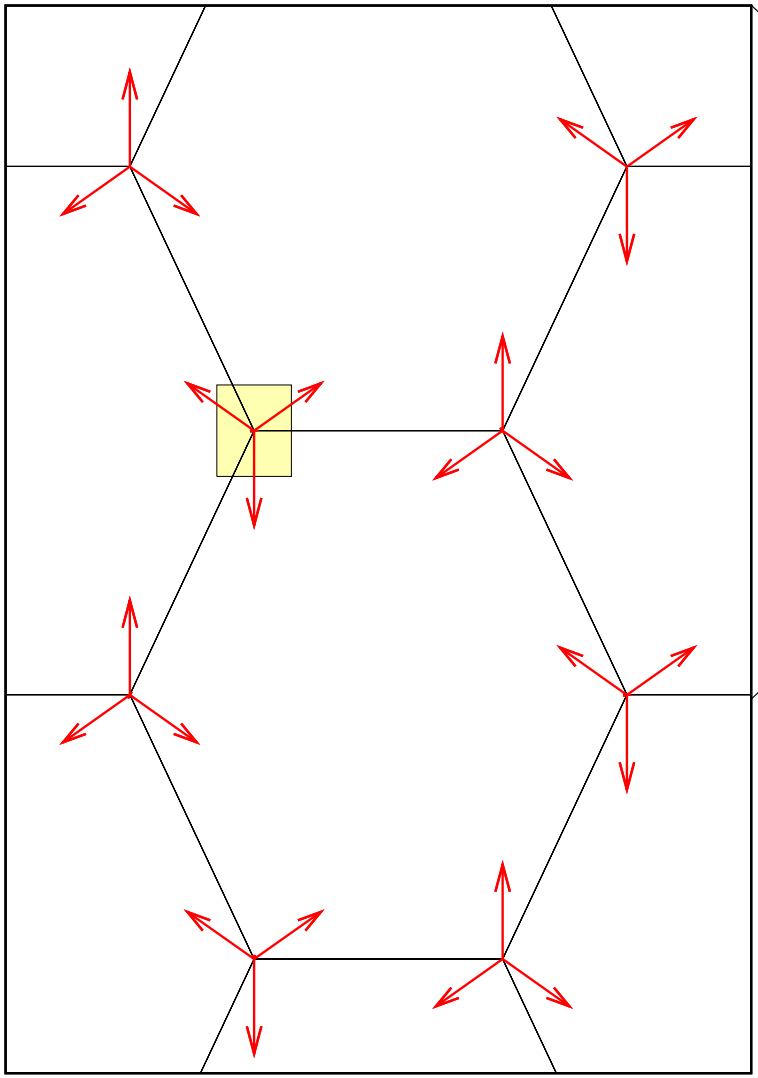

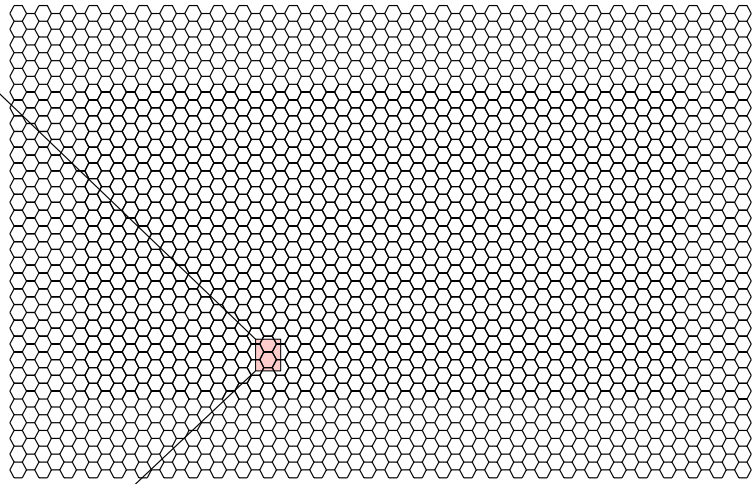

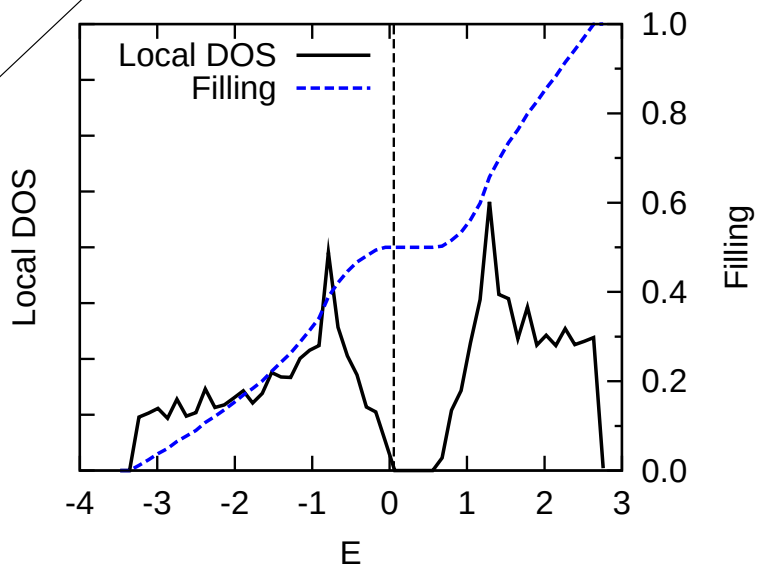

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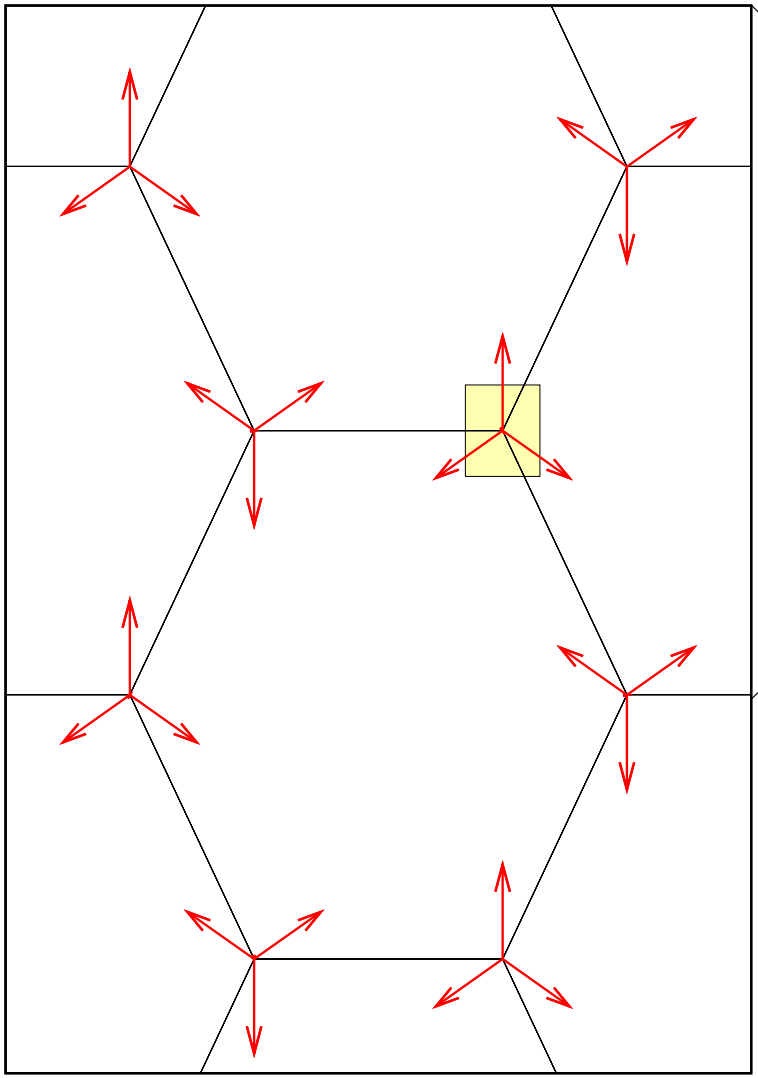

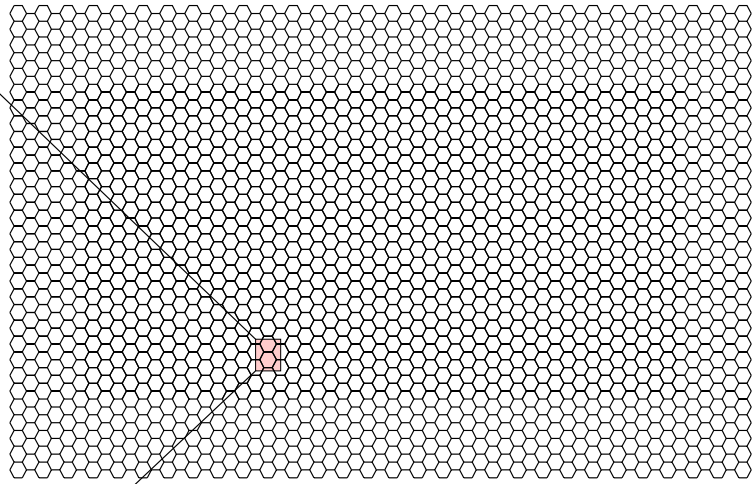

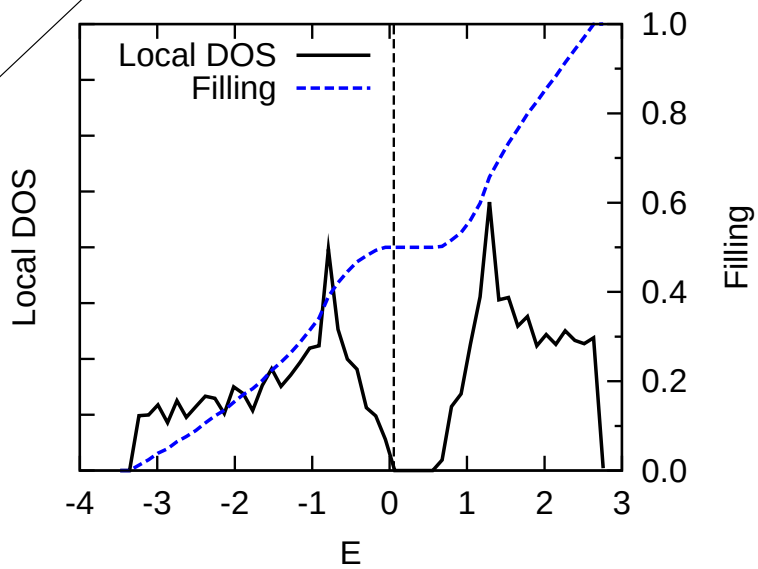

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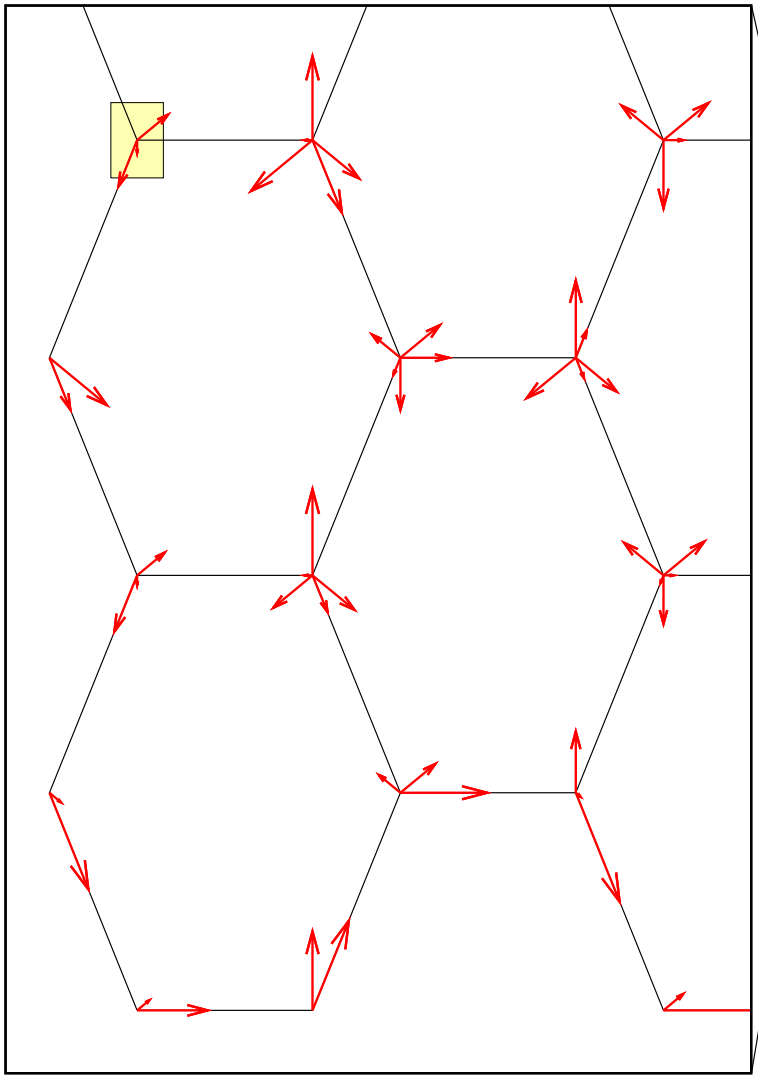

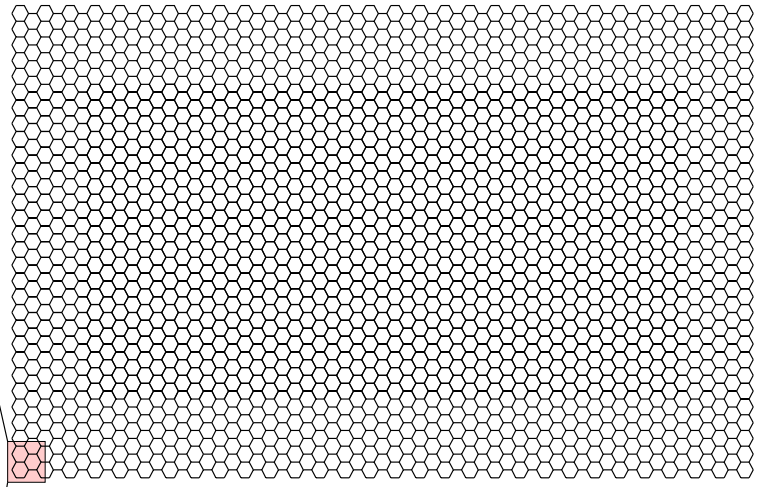

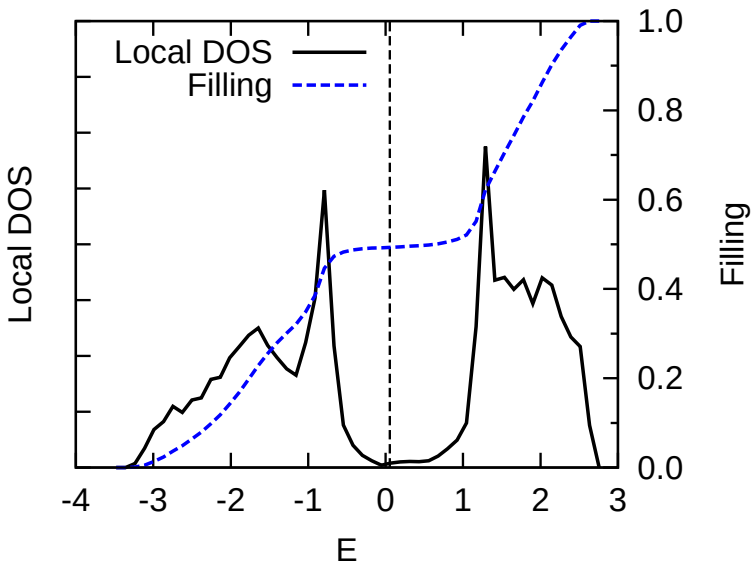

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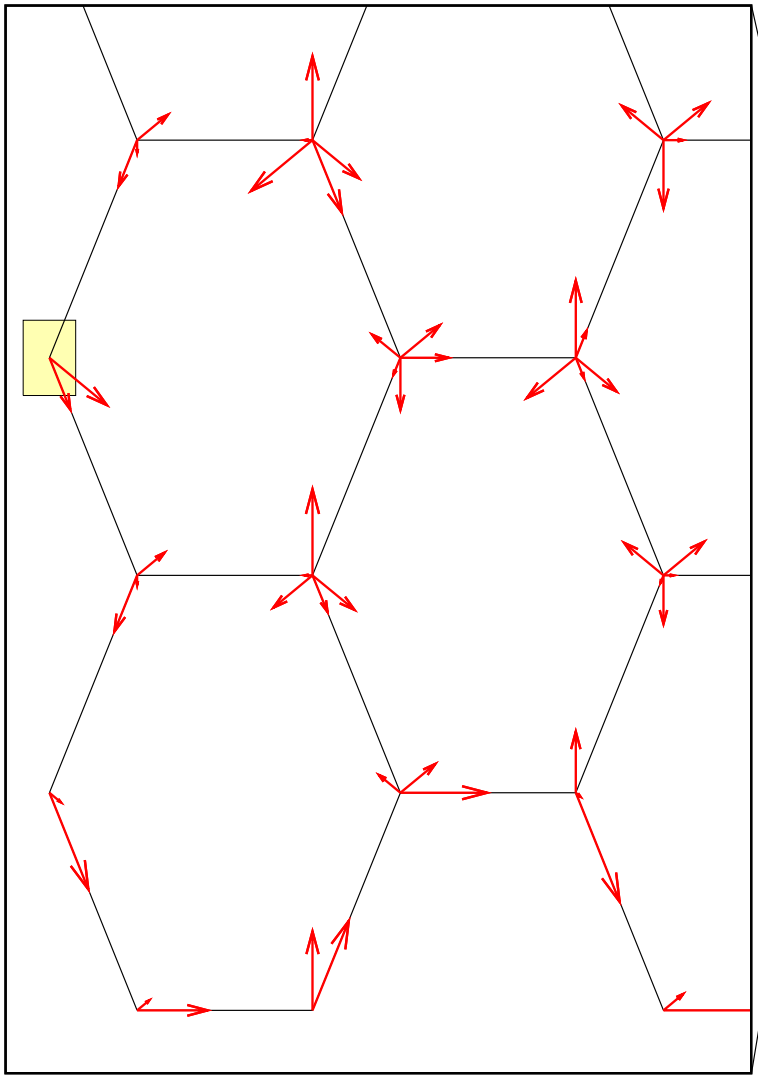

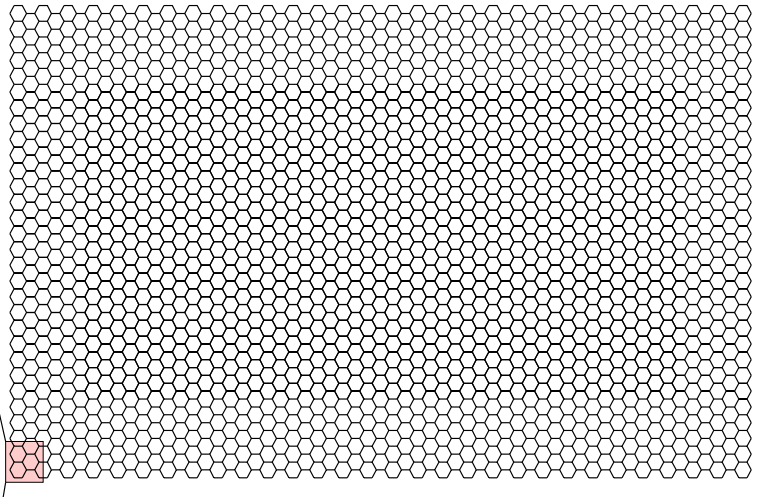

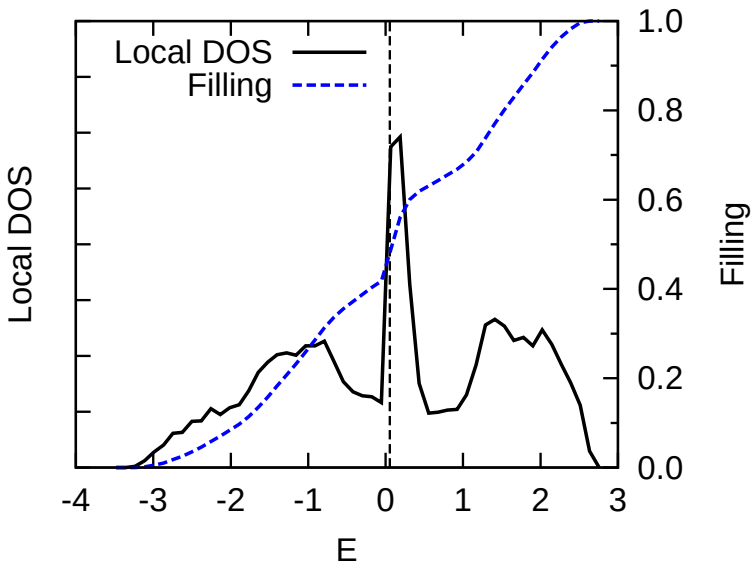

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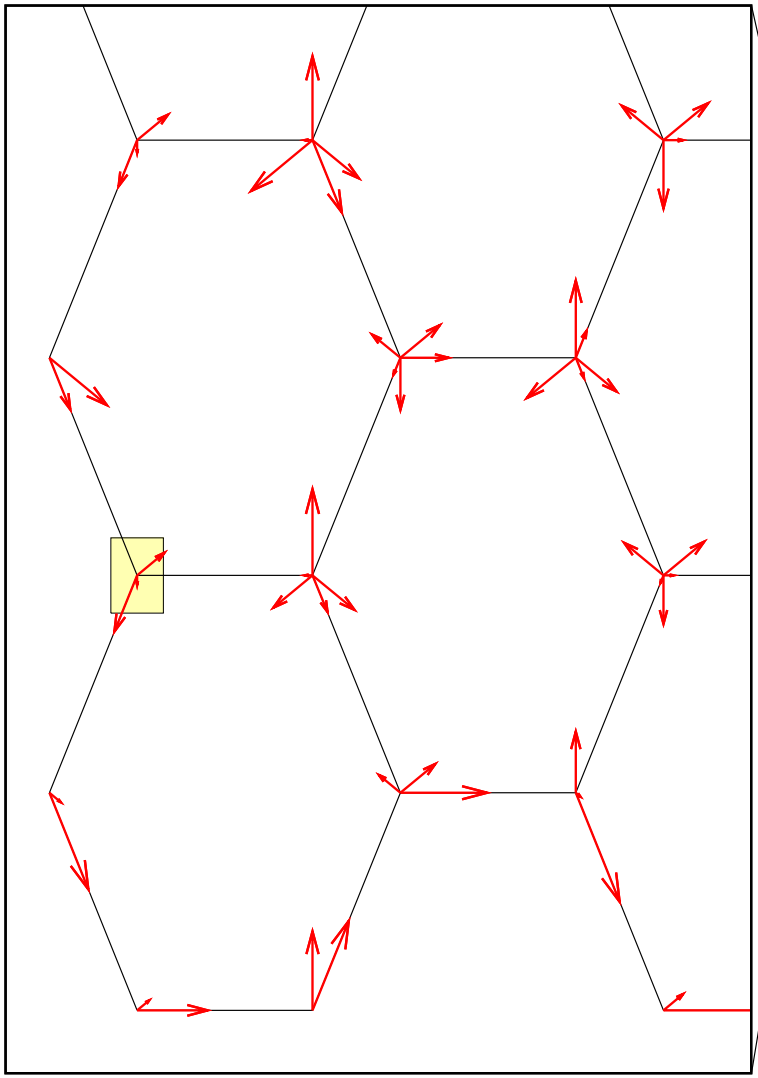

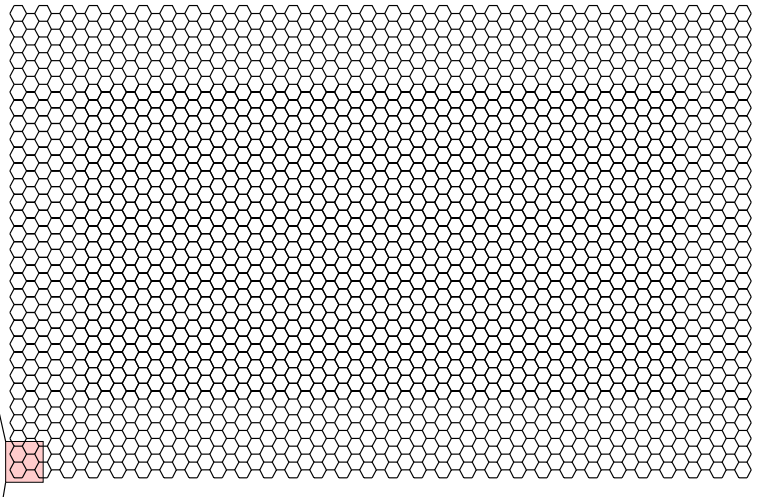

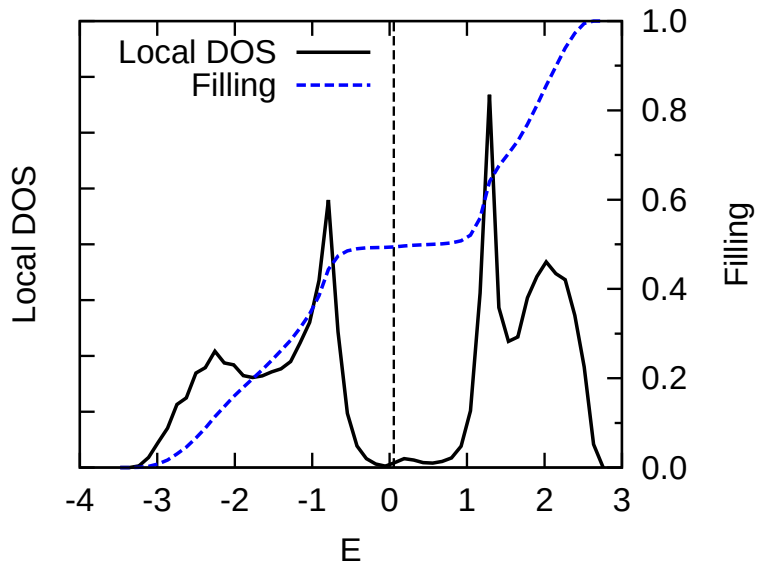

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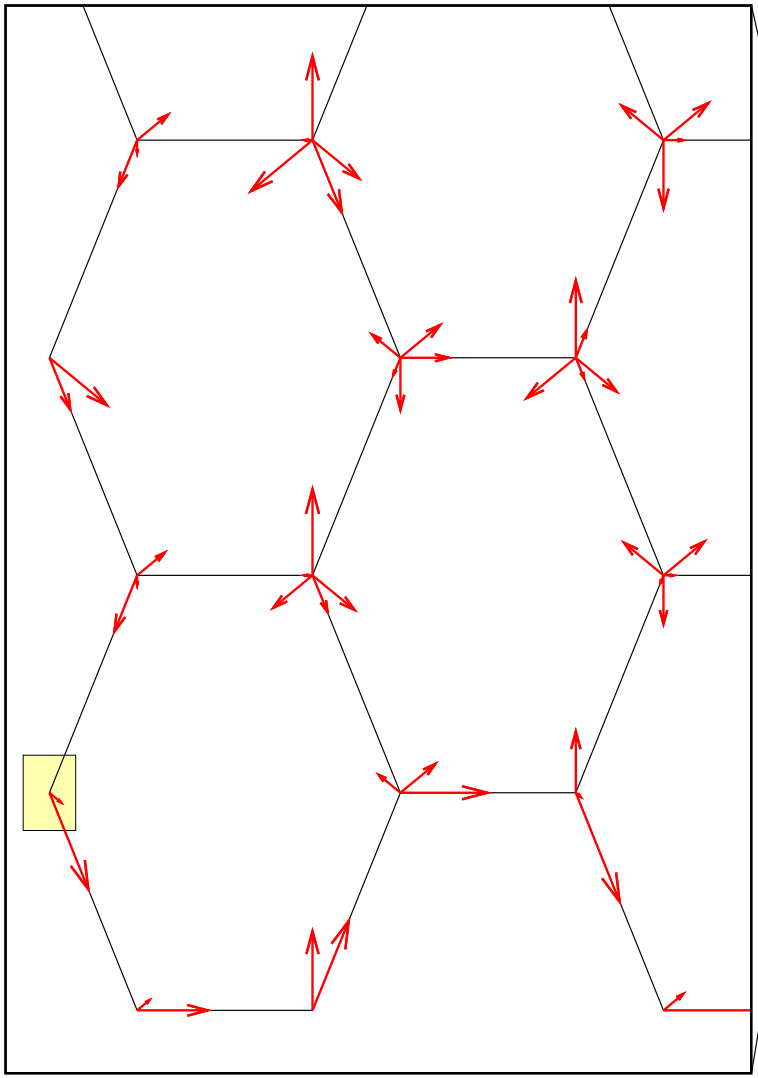

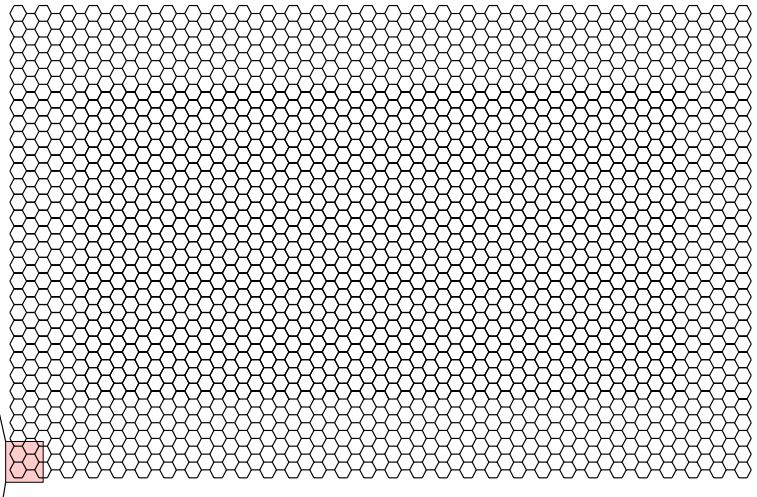

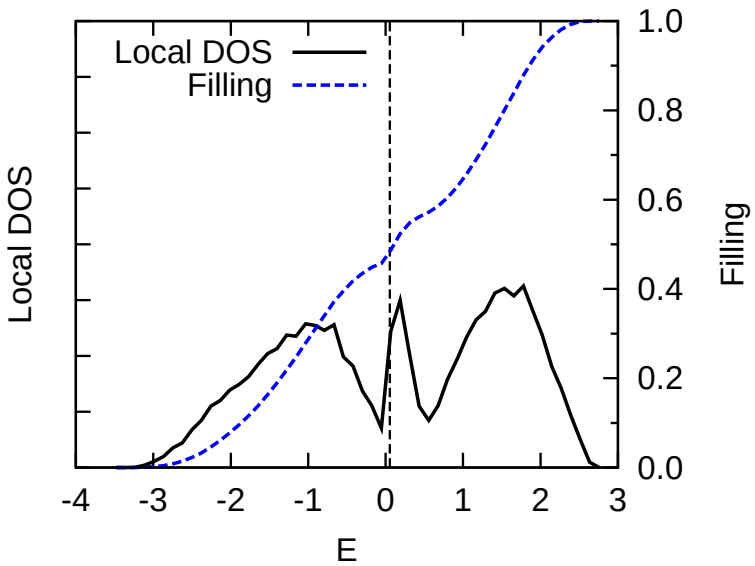

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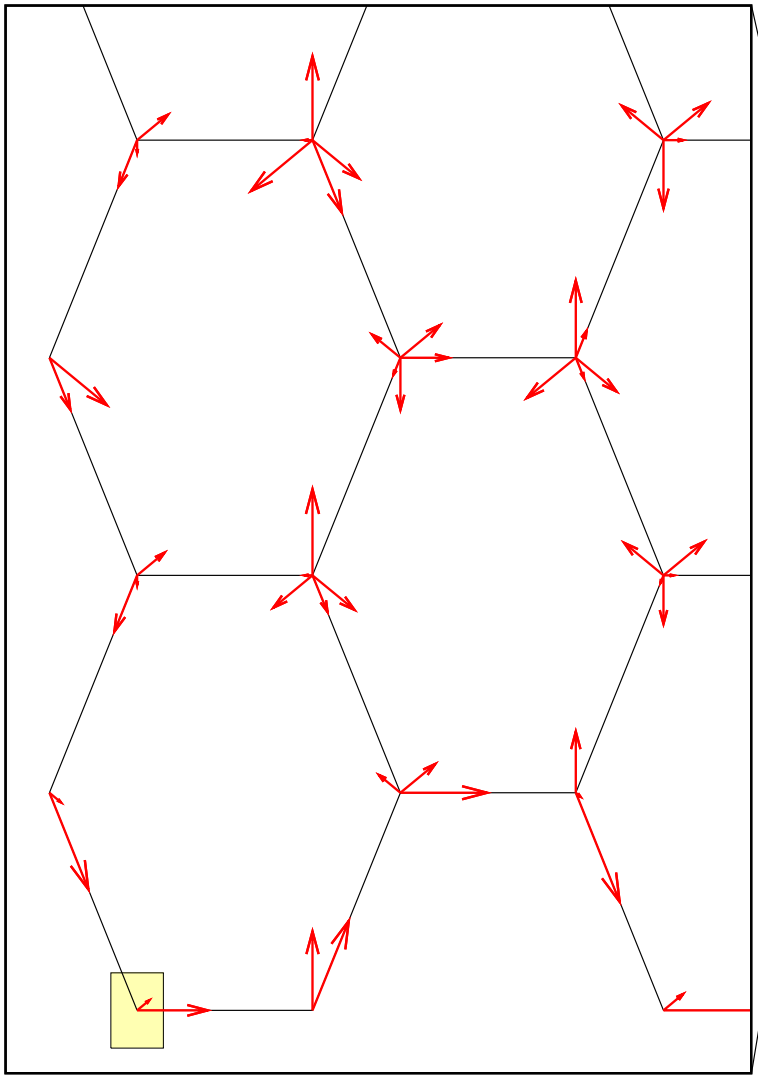

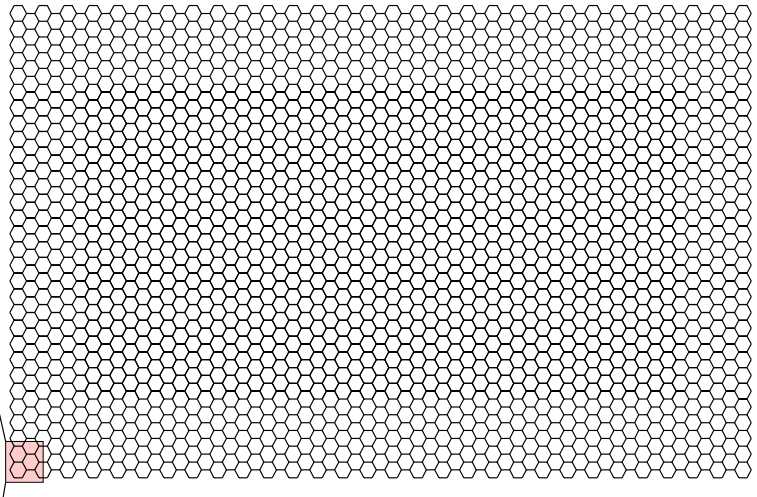

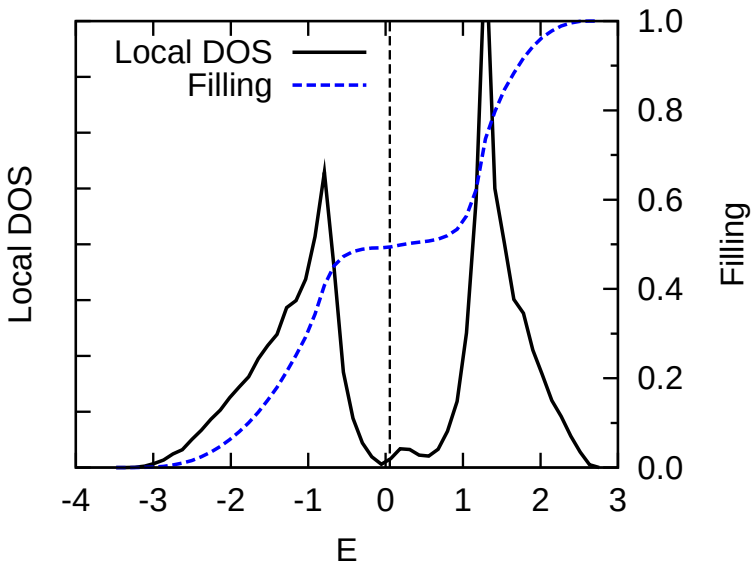

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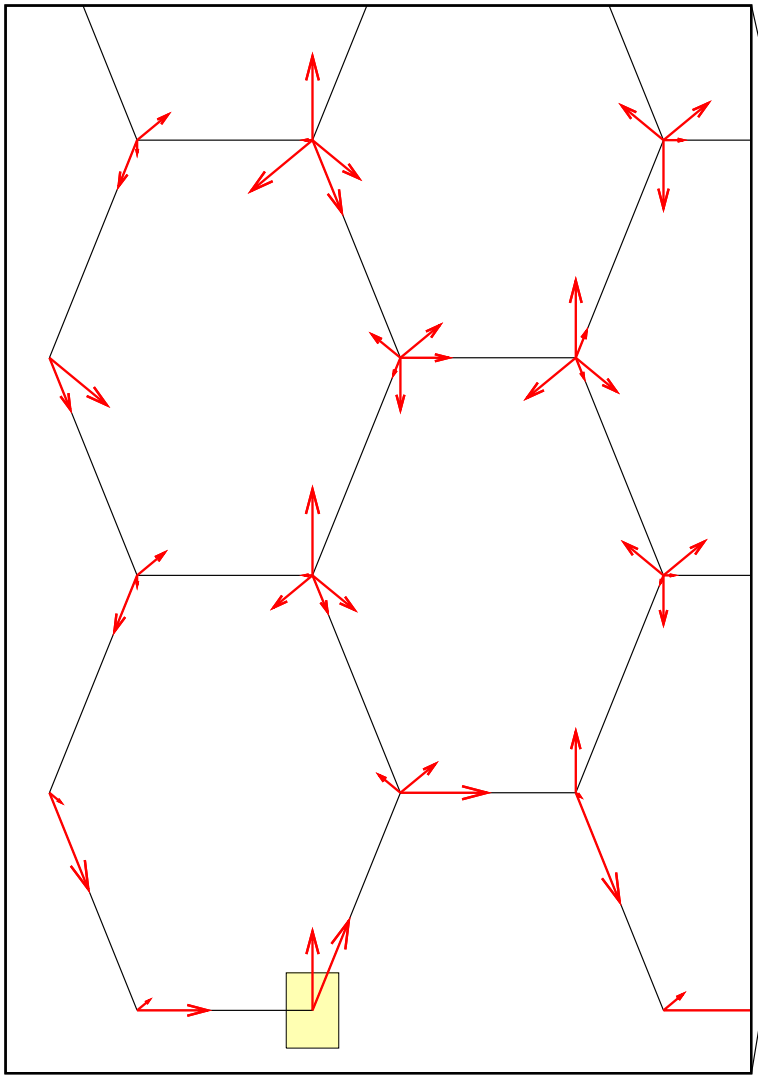

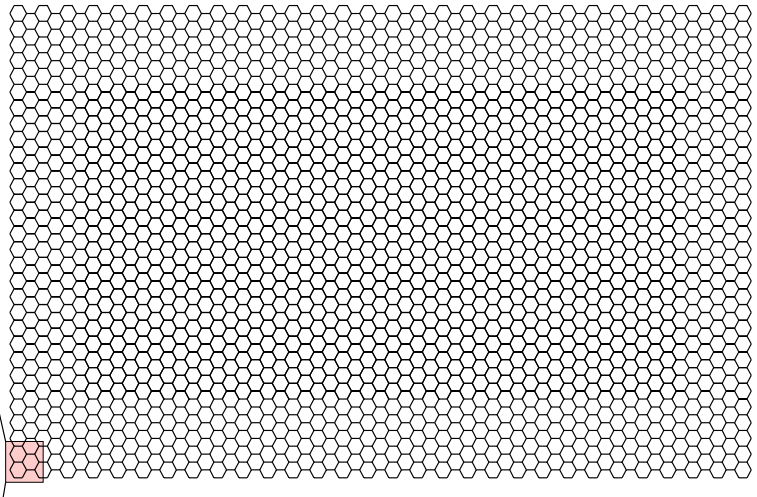

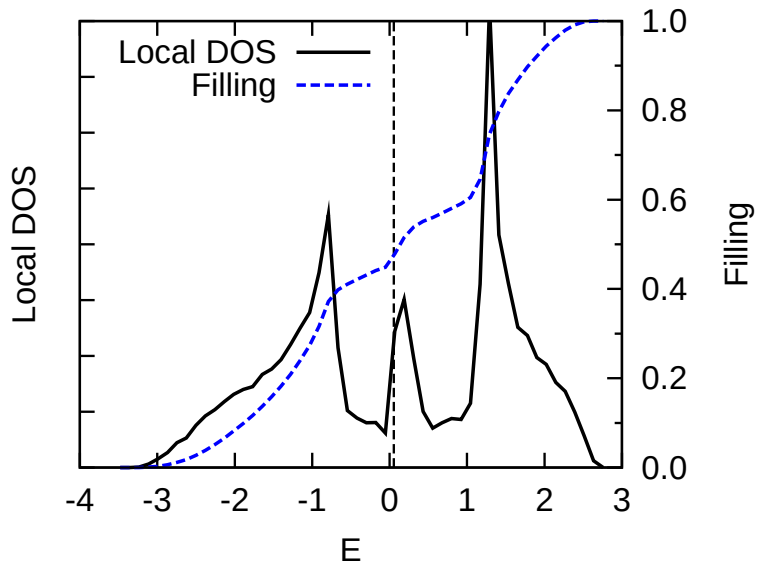

Summary

- Next-nearest-neighbor bonds on edge can cause a flux
  - Obscures observation of edge currents
- Modified Haldane
  - Not topologically protected
  - Infinitesimal $\mathcal{M}$ breaks chiral(inversion) symmetry
  - Anomalous Hall Effect
- Haldane
  - Topologically protected
  - Breaks time reversal symmetry
  - Quantum Anomalous Hall Effect

What's next?

- What role do interactions play? (Quantum Monte Carlo)