UCDAVIS

The FFLO Phase in Imbalanced Fermion Systems in 1-d

Theoretical Background
Experimental Motivation
Model and Method
Results - Uniform System
Results - Trapped System
Conclusions

G. G. Batrouni (INLN); M. H. Huntley (MIT); V. G. Rousseau (Leiden)
C. N. Varney, R. T. Scalettar (UC Davis)

Funding:
ARO Award W911NF0710576 with funds from the DARPA OLE Program

Theoretical Background

Usual Cooper pair: $(\, k\uparrow,\, -k\downarrow\,)$

What happens if $N_\uparrow \neq N_\downarrow$?

Mismatched Fermi surfaces: $k_{\mathrm{F,\,majority}} \neq k_{\mathrm{F,\,minority}}$

Breached Pair

- Superfluid of Cooper pairs $(\, k\uparrow,\, -k\downarrow\,)$ for $k < k_{\mathrm{F,\,minority}}$.
  coexists with normal fluid (of excess species).
- Pairs have zero momentum.
- Translationally invariant.

Fulde-Ferrell-Larkin-Ovchinnikov

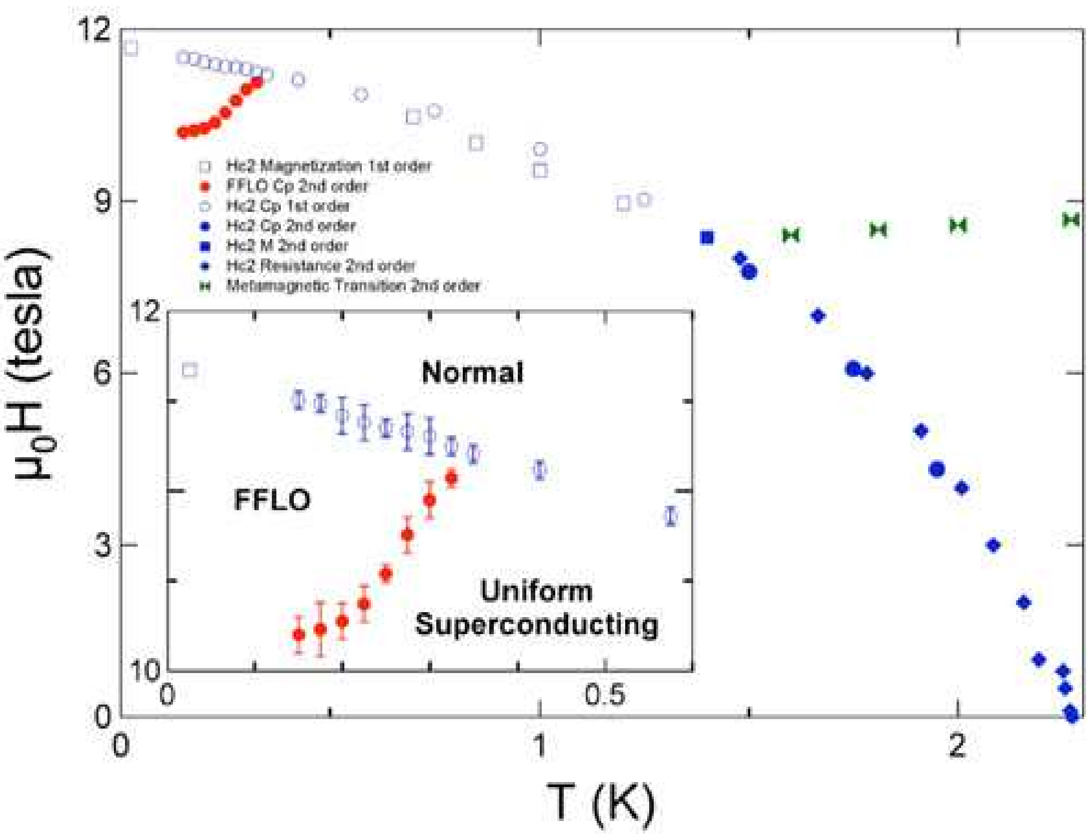
- Pairs have non-zero momentum $k_{\mathrm{F,\,majority}} - k_{\mathrm{F,\,minority}}$.
- Spatially inhomogeneous.
- Hard to see in CM systems.

Cold Atom Systems

- Two hyperfine states play role of spin up and down.
- One complication is role of trapping potential.

Experimental Motivation - Solid State

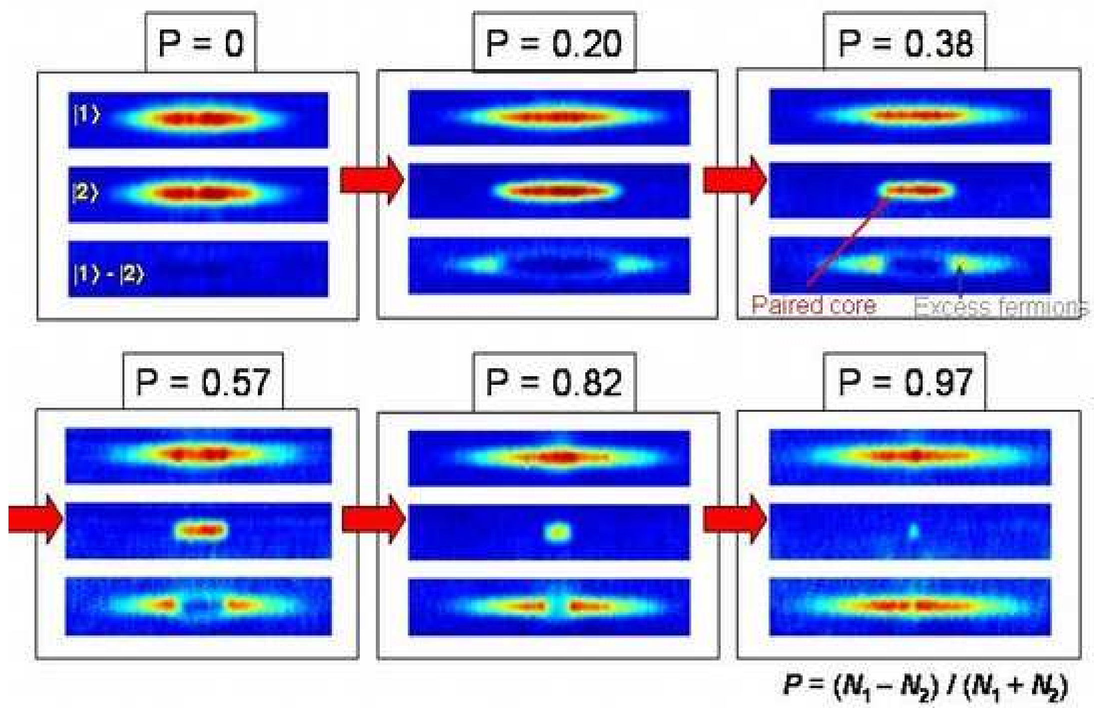
Forty years after its theoretical discussion, FFLO phase observed.
Heavy fermion system $CeCoIn_5$.
Requires very pure and strongly anisotropic single crystals.
Apply large field parallel to conducting planes.



H.A. Radovan *et al.*, Nature **425**, 51 (2003).

Experimental Motivation - Cold Atoms

Fermion ^{6}Li in hyperfine states $F = \frac{1}{2}, m_F = \pm\frac{1}{2}$.
Three dimensional, but highly elongated, traps.
Tunable interaction strength via Feschbach resonance.
Tunable relative $m_F = \pm\frac{1}{2}$ populations.

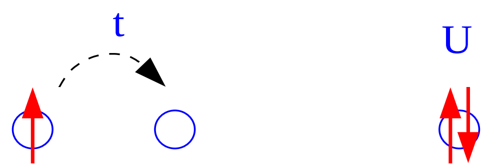


Core of system has uniform pairing ($n_1 - n_2 = 0$). Excess atoms sit at edge.
G.B. Partridge *et al.*, Science **311**, 503 (2006).
Also: M.W. Zwierlein *et al.*, Science **311**, 492 (2006); Nature **422**, 54 (2006).

The Attractive Fermion Hubbard Hamiltonian

$$H = -t \sum_{j,\sigma} (c^\dagger_{j\sigma} c_{j+1\sigma} + c^\dagger_{j+1\sigma} c_{j\sigma}) - |U| \sum_j n_{j\uparrow} n_{j\downarrow} + V_T \sum_j j^2 (n_{j\uparrow} + n_{j\downarrow})$$

Operators $c^\dagger_{i\sigma}$ ($c_{i\sigma}$) create (destroy) an electron of spin σ on site i.
Electron kinetic energy t; interaction energy U; Quadratic confining potential V_T.



Condensed matter: Two spin species $\sigma = \uparrow, \downarrow$.
Optically Trapped Atoms: Two hyperfine states "σ" $= 1, 2$.

Observables

$$\begin{aligned} G_\sigma(l) &= \langle c^\dagger_{j+l\,\sigma} c_{j\,\sigma} \rangle && \text{Fourier transform} : n_\sigma(k) \\ G_{\rm pair}(l) &= \langle \Delta_{j+l} \Delta^\dagger_j \rangle && \text{Fourier transform} : n_{\rm pair}(k) \\ \Delta_j &= c_{j2} c_{j1} \end{aligned}$$

Algorithm

Continuous time canonical ‘worm’ algorithm (Rombouts, Van Houcke, Pollet).

No discretization of imaginary time (no ‘Trotter’ errors).

Constant particle number.

Broken world lines (‘worms’) are propagated.

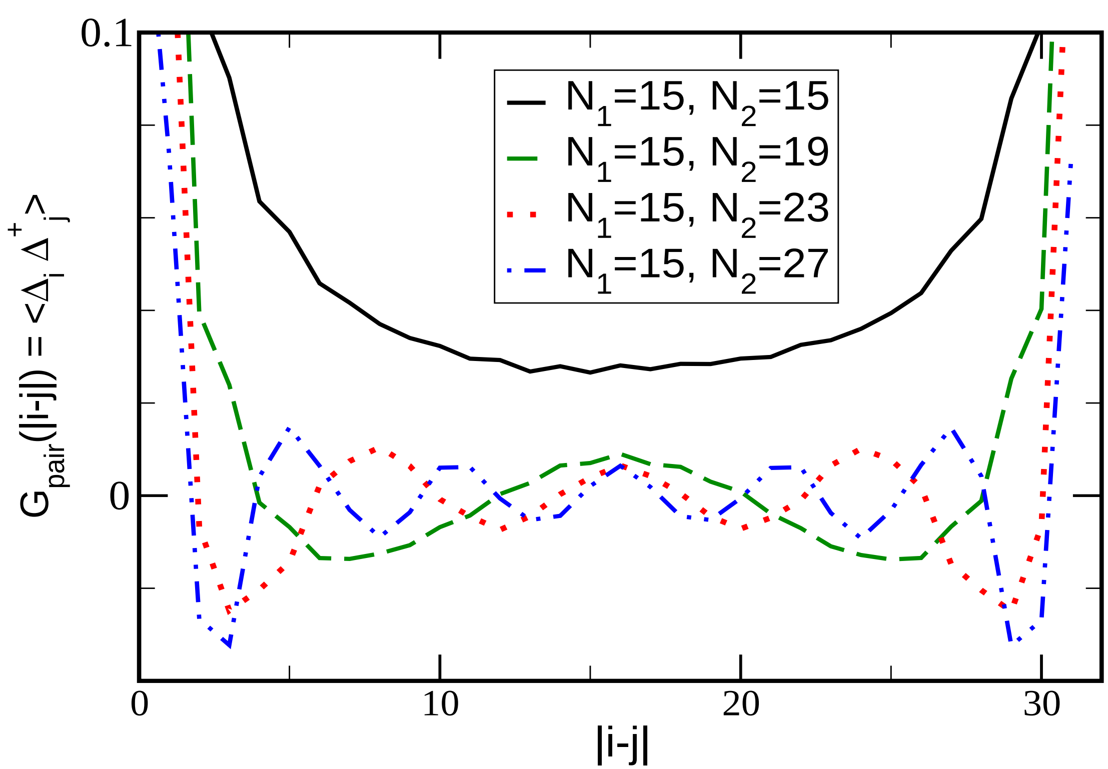
- Large moves through configuration space (short correlation times).
- Can measure non-local Greens functions.

Results for Uniform System

$$G_\sigma(l) = \langle c^\dagger_{j+l\,\sigma} c_{j\,\sigma} \rangle \qquad \text{Fourier transform}: n_\sigma(k)$$

$$G_{\rm pair}(l) = \langle \Delta_{j+l} \Delta^\dagger_j \rangle \qquad \text{Fourier transform}: n_{\rm pair}(k)$$

$$\Delta_j = c_{j2} c_{j1}$$

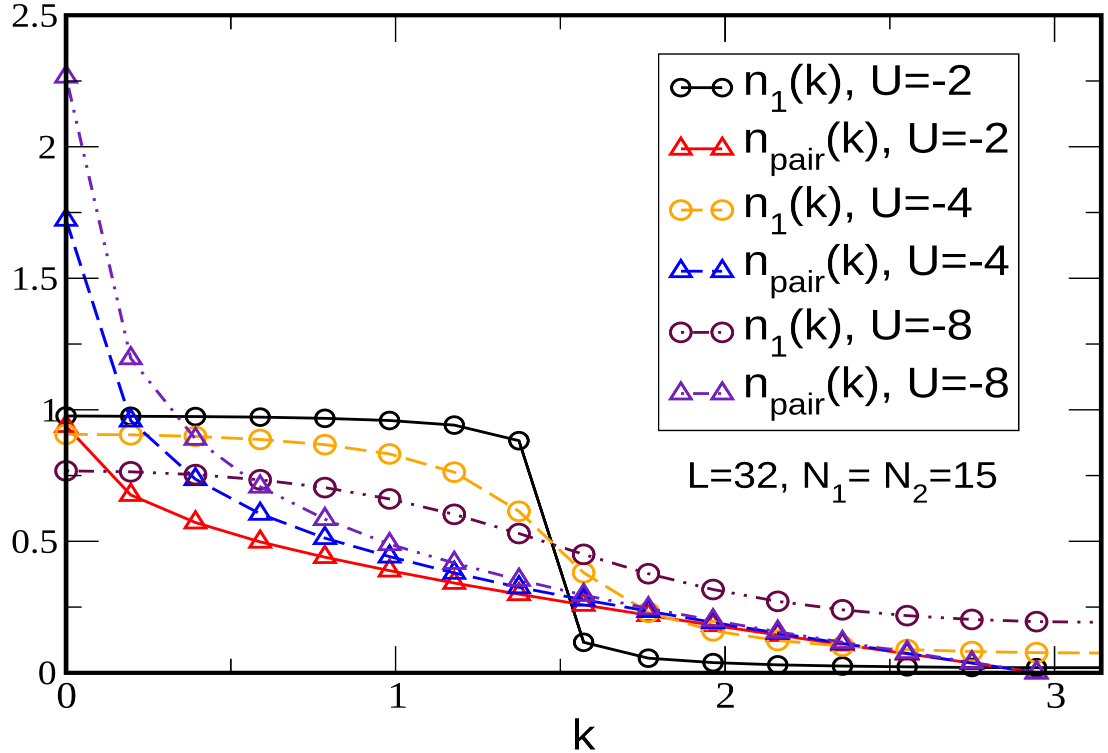


$U = -8$

$G_{\rm pair}(l)$ oscillates as $\cos(qr)$ with $q = k_{\rm F,\,majority} - k_{\rm F,\,minority}$ consistent with LO.

Begin analysis of $n_\sigma(k)$ and $n_{\rm pair}(k)$ by examining unpolarized case

- Weak coupling: $n_1(k) = n_2(k)$ is sharp.
- Strong coupling: $n_1(k) = n_2(k)$ rounded.
- $n_{\rm pair}(k)$ peaked at $k = 0$. Peak sharpens with $|U|$.



Polarized case

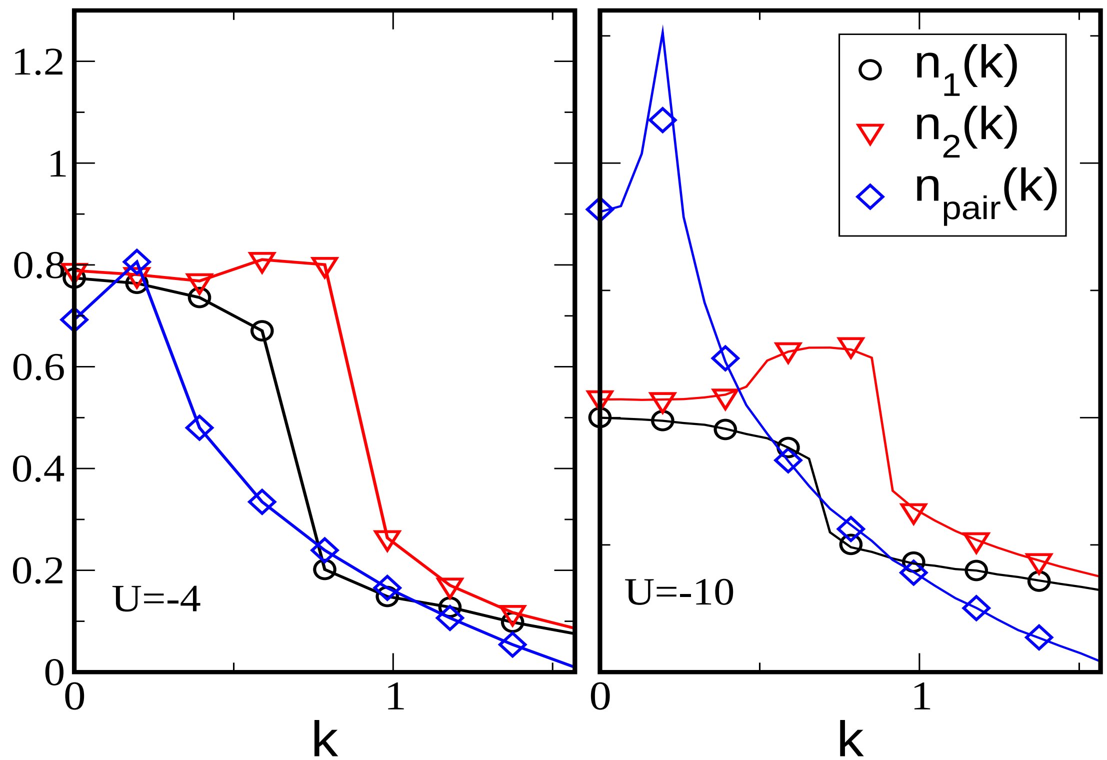
$n_{\text{pair}}(k)$ peaked at $k_{\text{F, majority}} - k_{\text{F, minority}}$ for all $|U|$.

Left panel: $U = -4$

Right panel: $U = -10$

Symbols: $N_1 = 7, N_2 = 9, L = 32$ sites, $\beta = 64$.

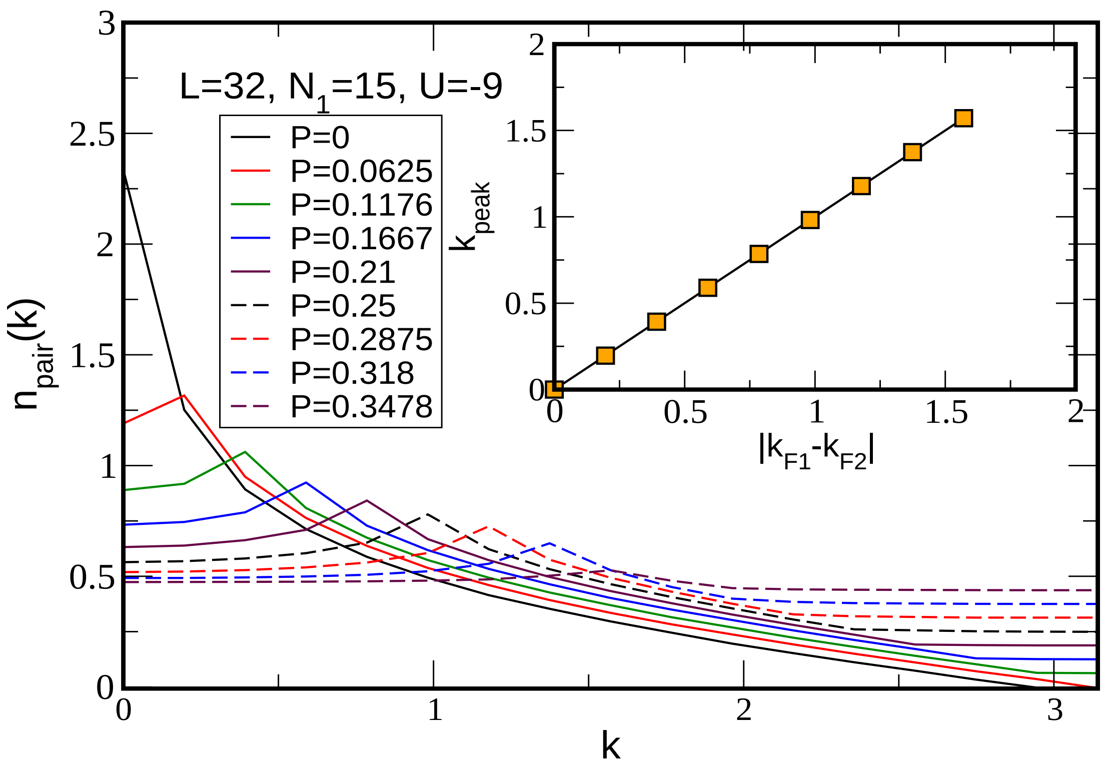
Lines: $N_1 = 21, N_2 = 27, L = 96$ sites, $\beta = 192$.



FFLO pairing no matter how large the polarization is made.

Have not seen "Clogston Limit".

Inset: Peak in $n_{\rm pair}(k)$ scales precisely as $k_{\rm F,\, majority} - k_{\rm F,\, minority}$.



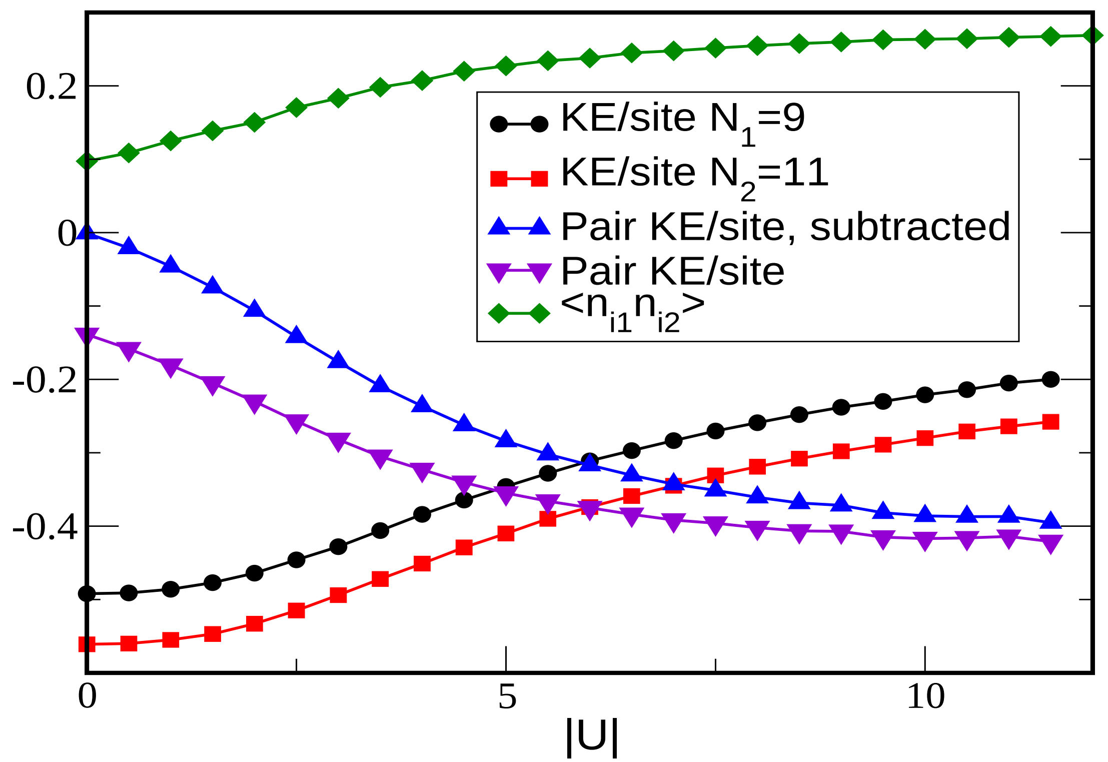
Kinetic Energy

As $|U|$ increases:

|Single particle KE| decreases

|Pair KE| increases

Hopping is increasingly "in pairs".



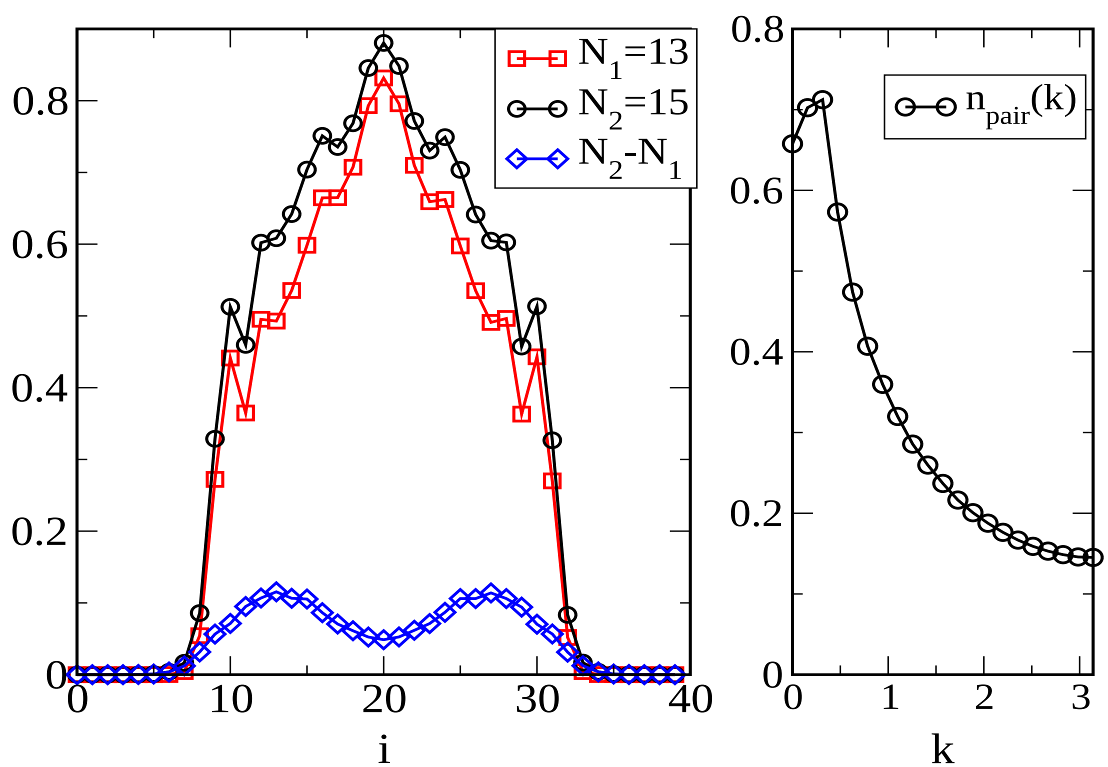
Cross-over occurs as double occupancy $\langle n_{i1} n_{i2} \rangle$ approaches saturation.

Results for Trapped System

Pronounced minimum in density difference at trap center.

$n_{\rm pair}(k)$ peaked at nonzero k. (System remains FFLO.)

$k_{\rm peak}$ consistent with value of local polarization at trap center.



Conclusions

Clear evidence for FFLO phase in Imbalanced $d = 1$ Attractive Hubbard Model

Uniform System

- Spatially oscillating $G_{\mathrm{pair}}(r)$
- $n_{\mathrm{pair}}(k)$ peaked at $k = k_{\mathrm{F,majority}} - k_{\mathrm{F,minority}}$
- No Clogston limit

Trapped System

- Deep minimum in local polarization at trap center.
- System retains signature of FFLO phase: $n_{\mathrm{pair}}(k)$ peaked at $k \neq 0$.

G. G. Batrouni (INLN); M. H. Huntley (MIT); V. G. Rousseau (Leiden);
C. N. Varney, R. T. Scalettar (UC Davis)

G. G. Batrouni, M. H. Huntley, V. G. Rousseau, RTS (Phys. Rev. Lett., to appear)

Funding:
ARO Award W911NF0710576 with funds from the DARPA OLE Program